ON AN ADJUSTMENT FOR THE PLANE GRATING SIMILAR TO ROWLAND'S METHOD FOR THE CONCAVE GRATING.[1]

By CARL BARUS.

(*Read April 24, 1909.*)

1. *Apparatus.*—The remarkable refinement which has been attained (notably by Mr. Ives and others) in the construction of celluloid replicas of the plane grating, makes it desirable to construct a simple apparatus whereby the spectrum may be shown and the measurement of wave-length made, in a way that does justice to the astonishing performance of the grating. We have, therefore, thought it not superfluous to devise the following inexpensive contrivance, in which the wave-length is strictly proportional to the shift of the carriage at the eye-piece; which for the case of a good 2-meter scale divided into centimeters, admits of a measurement of wave-length to a few Ångström units and with a millimeter scale should go much further.

Observations are throughout made on both sides of the incident rays and from the mean result most of the usual errors should be eliminated by symmetry.

In Fig. 1, A and B are two double slides, like a lathe bed, 155 cm. long and 11 cm. apart, which happened to be available for optical purposes, in the Laboratory. They were therefore used, although single slides at right angles to each other, similar to Rowland's, would have been preferable. The carriages C and D, 30 cm. long, kept at a fixed distance apart by the rod aRb, are in practice a length of $\frac{1}{4}$-inch gas pipe, swivelled at a and b, 169.4 centimeters apart, and capable of sliding right and left and to and fro, normally to each other.

[1] The investigations in this paper were undertaken throughout in conjunction with my son, Mr. Maxwell Barus; but it seemed advisable that I should undertake the publication in these Proceedings myself, with the present acknowledgment.

The swivelling joint which functioned excellently, is made very simply of $\frac{1}{4}$-inch gas pipe T's and nipples, as shown in Fig. 2. The lower nipple N is screwed tight into the T, but all but tight into the carriage D, so that the rod ab turns in the screw N, kept oiled. Similarly the nipple N'' is either screwed tight into the T (in one

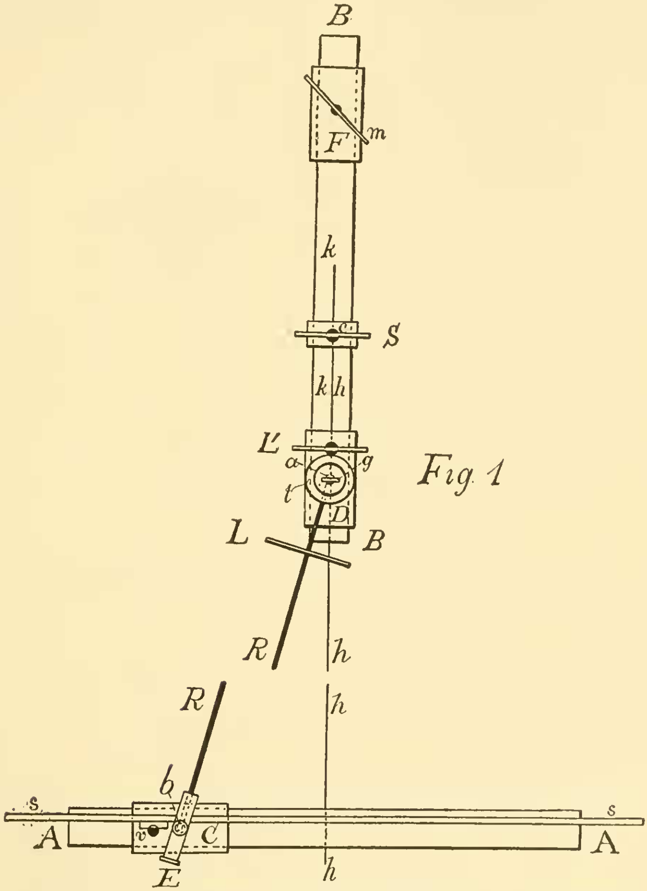


FIG. 1. Plan of apparatus. *AA, BB,* slides; *C, D,* carriages; *R,* connecting rod.

method, revoluble grating), or all but tight (in another method, stationary grating), so that the table *tt*, which carries the grating *g* may be fixed while the nipple N'' swivels in the T. Any ordinary laboratory clamp K and a similar one on the upright c (screwed into the carriage S) secures a small rod k for this purpose. Again a hole may be drilled through the standards at K and c and provided with set screws to fix a horizontal rod k or check. The rod k should be long enough to similarly fix the standard on the slide S carrying the slit and be prolonged further toward the rear to carry the flame or Geissler tube apparatus. The table tt is revoluble on a brass rod fitting within the gas pipe, which has been slotted across so that the conical nut M may hold it firmly. The axis passes through the middle of the grating, which is fastened centrally to the table tt with the usual tripod adjustment.

2. *Single Focusing Lens in Front of Grating.*—I shall describe three methods in succession, beginning with the first. Here a large lens L, of about 56 cm. focal distance and about 10 cm. in diameter, is placed just in front of the grating, properly screened and throwing an image of the slit S upon the cross-hairs of the eye-piece E, the line of sight of which is always parallel to the rod ab, the end b swivelled in the carriage C, as stated (see Fig. 2). An ordinary lens of 5 to 10 cm. focal distance, with an appropriate diaphragm, is adequate and in many ways preferable to stronger eye-pieces. The slit S, carried on its own slide and capable of being clamped to c when necessary, as stated, is additionally provided with a long rod hh lying underneath the carriage, so that the slit S may be put accurately in focus by the observer at C. F is a carriage for the mirror or the flame or other source of light whose spectrum is to be examined; or the source may be adjustable on the rear of the rod by which D and S are locked together.

Finally the slide AB is provided with a scale ss and the position of the carriage C read off by aid of the vernier v. A good wooden scale graduated in centimeters happened to be available, the vernier reading to within one millimeter. For more accurate work a brass scale in millimeters with an appropriate vernier should of course be used.

Eye-piece E, slit S, flame F, etc., may be raised and lowered by the split tube devise shown as at M and M' in Fig. 2.

3. *Adjustments.*—The first general test which places slit, grating and its spectra and the two positions of the eye-piece in one plane,

is preferably made with a narrow beam of sunlight, though lamp-light suffices in the dark. Thereafter let the slit be focused with the eye-piece on the right marking the position of the slit; next focus the slit for the eye-piece on the left; then place the slit mid-way between these positions and now focus by slowly rotating the grating. The slit will then be found in focus for both positions

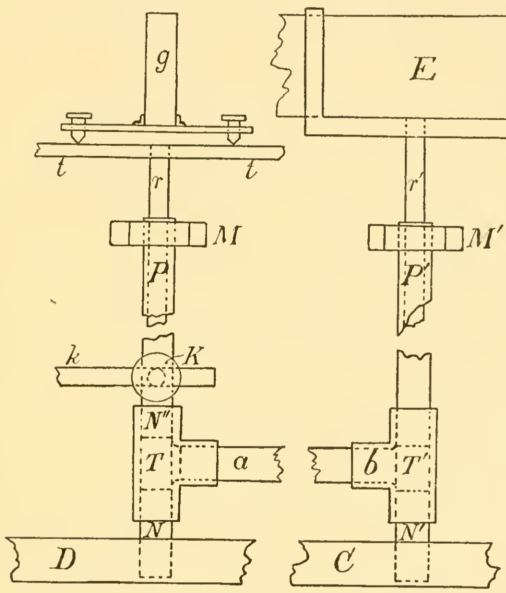


Fig. 2.

FIG. 2. Elevation of the grating (g) and the eyepiece (E) standards.

and the grating which acts as a concave lens counteracting L will be symmetrical with respect to both positions.

Let the grating be thus adjusted when fixed normally to the slide B or parallel to A. Then for the first order of the spectra the wave-length $\lambda = d \sin \theta$, where d is the grating space and θ the angle of diffraction. The angle of incidence i is zero.

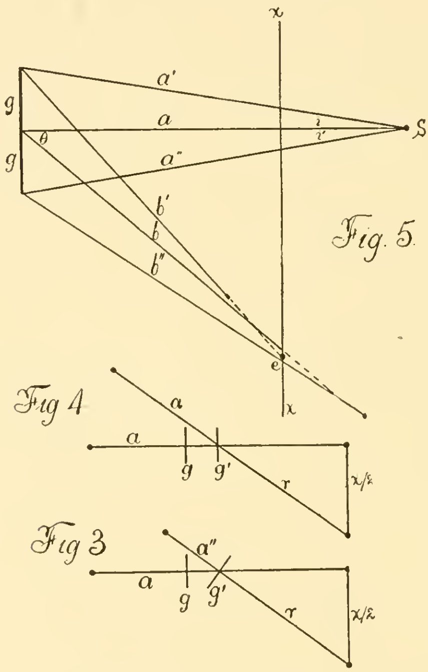
Again let the grating, adjusted for symmetry, be free to rotate with the rod ab. Then θ is zero and $\lambda = d \sin i$.

In both cases however if $2x$ be the distance apart of the carriage C, measured on the scale ss, for the effective length of rod $ab = r$ between axis and axis,

$$\lambda = dx/r \quad \text{or} \quad (d/2r)2x,$$

so that in either case λ and x are proportional quantities.

The whole spectrum is not however clearly in focus at one time, though the focusing by aid of the rod hh is not difficult. For extreme positions a pulley adjustment, operating on the ends of h



FIGS. 3, 4, 5. Diagrams.

is a convenience, the cords running around the slide AA. In fact if the slit is in focus when the eye-piece is at the center ($\theta = 0$, $i = 0$) at a distance a from the grating, then for the fixed grating, Fig. 4,

$$a' = a\frac{r^2}{r^2 - x^2},$$

where a' is the distance between grating and slit for the diffraction corresponding to x. Hence the focal distance of the grating regarded as a concave lens is $f' = ar^2/x^2$. For the fixed grating and a given color, it frequently happens that the undeviated ray and the diffracted rays of the same color are simultaneously in focus, though this does not follow from the equation.

Again for the rotating grating, Fig. 3, if a'' is the distance between slit and grating

$$a'' = a\frac{r^2 - x^2}{r^2},$$

so that its focal distance is

$$f'' = a\frac{r^2 - x^2}{x^2},$$

It follows also that $a' \times a'' = a^2$. For $a = 80$ cm. and sodium light, the adjustment showed roughly $f' = 650$ cm., $f'' = 570$, the behavior being that of weak concave lenses. The same $a = 80$ cm. and sodium light showed furthermore $a' = 91$ and $a'' = 70.3$.

Finally there is a correction needed for the lateral shift of rays, due to the fact that the grating film is enclosed between two moderately thick plates of glass (total thickness $t = .99$ cm.) of the index of refraction n. This shift thus amounts to

$$e = \frac{tx}{r}\left(\frac{1}{\sqrt{1 - x^2/r^2}} - \frac{1}{\sqrt{n^2 - x^2/r^2}}\right)\frac{b}{a}.$$

But since this shift is on the rear side of the lens L, its effect on the eye-piece beyond will be (if f is the principal focal distance and b the conjugate focal distance between lens and eye-piece, remembering that the shift must be resolved parallel to the scale ss)

$$e = \frac{tx}{r}\left(\frac{1}{\sqrt{1 - x^2/r^2}} - \frac{1}{\sqrt{n^2 - x^2/r^2}}\right)\left(\frac{b}{f} - 1\right),$$

where the correction e is to be added to $2x$, and is positive for the rotating grating and negative for the stationary grating.

Hence in the mean values of $2x$ for stationary and rotating grating the effect of e is eliminated. For a given lens at a fixed distance from the eye-piece $(b/f - 1)$ is constant.

4. *Data for Single Lens in Front of Grating.*—In conclusion we select a few results taken at random from the notes.

Grating	Line.	Observed $2x'$.	Shift.	Corrected $2x$.
Stationary	C	132.60	—.26	132.34
	D_2	118.90	—.23	118.67
	F	98.23	—.19	98.04
	Hydrogen Violet	87.87	—.16	87.71
Rotating	C	132.10	+.26	132.36
	D_2	118.45	.23	118.68
	F	97.90	.19	98.09
	H. Violet	87.50	.16	87.66

The real test is to be sought in the coresponding values of $2x$ for the stationary and rotating cases, and these are very satisfactory, remembering that a centimeter scale on wood and a vernier reading to millimeters only was used for measurement.

5. *Single Focusing Lens Behind the Grating.*—The lens L', which should be achromatic, is placed in the standard behind g. The light which passes through the grating is now convergent, whereas it was divergent in § 2. Hence the focal points at distances a', a'' lie in front of the grating; but in other respects the conditions are similar but reversed. Apart from signs, for the stationary grating

$$a' = a\frac{r^2 - x^2}{r^2},$$

and for the rotating grating

$$a'' = a\frac{r^2}{r^2 - x^2}.$$

The correction for shift loses the factor $(b/f - 1)$ and becomes

$$e = \frac{tx}{r}\left(\frac{1}{\sqrt{1 - x^2/r^2}} - \frac{1}{\sqrt{n^2 - x^2/r^2}}\right).$$

As intimated, it is negative for the rotating grating and positive for the stationary grating. It is eliminated in the mean values.

6. *Data. Single Lens Behind the Grating.*—An example of the results will suffice. Different parts of the spectrum require focusing.

Grating.	Line.	$2x'$	Shift.	$2x$
Stationary	D_2	118.40	+.13	118.53
Rotating	D_2	118.65	—.13	118.52

The values of $2x$, remembering that a centimeter scale was used, are again surprisingly good. The shift is computed by the above equation. It may be eliminated in the mean of the two methods. The lens L' may be more easily and firmly fixed than L.

7. *Collimator Method.*—The objection to the above single-lens methods is the fact that the whole spectrum is not in sharp focus at once. Their advantage is the simplicity of the means employed. If a lens at L' and at L are used together, the former as a collimator (achromatic) and with a focal distance of about 50 cm., and the latter (focal distance to be large, say 150 cm.) as the objective of a telescope, all the above difficulties disappear and the magnification may be made even excessively large. The whole spectrum is brilliantly in focus at once and the corrections for the shift of lines due to the plates of the grating vanish. Both methods for stationary and rotating gratings give identical results. The adjustments are easy and certain, for with sunlight (or lamplight in the dark) the image of the slit may be reflected back from the plate of the grating on the plane of the slit itself, while at the same time the transmitted image may be equally sharply adjusted on the focal plane of the eye-piece. It is therefore merely necessary to place the plane of spectra horizontal. Clearly a' and a'' are all infinite.

In this method the slide S and D are clamped at the focal distance apart, so that flame, etc., slit, collimator lens and grating move together. The grating may or may not be revoluble with the lens L on the axis a.

8. *Data for the Collimator Method.*—The following data chosen at random may be discussed. The results were obtained at different times and under different conditions. The grating nominally contained about 15,050 lines per inch. The efficient rod length ab was $R = 169.4$ cm. Hence if $1/C = 15{,}050 \times .3937 \times 338.8$, the wavelength $\lambda = C \cdot 2x$ cm.

Grating.	Lines.	$2x'$	$2x$
Stationary	D_2	118.30	118.19
Rotating	D_2	118.08	118.19
Stationary	D_2	118.27	118.16
Rotating	D_2	118.05	118.16

Rowland's value of D_2 is 58.92×10^{-6} cm.; the mean of the two values of $2x$ just stated will give 58.87×10^{-6} cm. The difference may be due either to the assumed grating space, or to the value of R inserted, neither of which were reliable absolutely to much within .1 per cent.

Curious enough an apparent shift effect remains in the values of $2x$ for stationary and rotating grating, as if the collimation were imperfect. The reason for this is not clear, though it must in any case be eliminated in the mean result. Possibly the friction involved in the simultaneous motion of three slides is not negligible and may leave the system under slight strain equivalent to a small lateral shift of the slit.

9. *Discussion.*—The chief discrepancy is the difference of values for $2x$ in the single lens system (for D_2, 118.7 and 118.5 cm., respectively) as compared with a double lens system (for D_2, 118.2 m.) amounting to .2 to .4 per cent. For any given method this difference is consistently maintained. It does not, therefore, seem to be mere chance.

We have for this reason computed all the data involved for a fixed grating 5 cm. in width, in the two extreme positions, Fig. 5, the ray being normally incident at the left hand and the right hand edge respectively for the method of § 6. The meaning of the symbols is clear from Fig. 5, S being the virtual source, g the grating, e the diffraction conjugate focus of S for normal incidence, so that $b = r$ is the fixed length of rod carrying grating and eye-piece. It is almost sufficient to assume that all diffracted rays b' to b'' are directed towards e, in which case equations (1) would hold; but this will not bring out the divergence in question. They were therefore not used. Hence the following equations (2) to (5) successively apply where d is the grating space.

(1) $\cot\theta' = (b/g + \sin\theta)/\cos\theta;\quad \cot\theta'' = (b/g - \sin\theta)/\cos\theta;$

(2) $a = b/\cos^2\theta;\quad a' = a'' = \sqrt{g^2 + a^2};$

(3) $\sin i' = \sin i'' = g/a';$

(4) $-\sin i' + \sin(\theta + \theta') = \lambda/d;\quad \sin\theta = \lambda/d;$
$\sin i'' + \sin(\theta - \theta'') = \lambda/d;$

(5) $\cos^2 i'/a' = \cos^2(\theta + \theta')/b';\quad \cos^2 i''/a'' = \cos^2(\theta - \theta'')/b''.$

Since θ, g, λ, d, b, are given θ' and θ'' are found in equation (4), apart from signs. If δ_1 and δ_1'' be the distance apart of the projections of the extremities of b' and b, b and b'', respectively, on the line x,

$$(6)\qquad \begin{aligned} \delta_1' &= g + (b - b')\sin\theta - b'\sin i' \\ \delta_1'' &= g + (b'' - b)\sin\theta - b''\sin i'' \end{aligned}$$

If δ_2' and δ_2'' be the distance apart of the intersections of the prolongation of b' and b, b and b'', respectively, with the line x,

$$(7)\qquad \begin{aligned} \delta_2' &= \sin(\theta + \theta')\,(b\cos\theta/\cos(\theta + \theta') - b') \\ \delta_2'' &= \sin(\theta - \theta'')\,(b'' - b\cos\theta/\cos(\theta - \theta'')) \end{aligned}$$

Given $b = 169.4$ cm., $\theta = 20°\ 22'$, about for sodium, $g = 5$ cm., the following values are obtained:

$$\begin{array}{lll} \theta' = 1°\ 36', & a = 192.7 \text{ cm.}, & b' = 166.0 \text{ cm.}, \\ \theta'' = 1°\ 34', & a' = a'' = 192.8 \text{ cm.}, & r = b = 169.4 \text{ cm.}, \\ i' = i'' = 1°\ 30', & & b'' = 172.4 \text{ cm.}, \end{array}$$

whence

$$\delta_1' = 1.92 \text{ cm.}, \quad \delta_2'' = 1.74 \text{ cm.}$$

These limits are surprisingly wide. If, however, they should be quite wiped out on focusing, for any group of rays and symmetrical observations on the two sides of the apparatus, this would be no source of discrepancy. The effect of focusing the two parts of the grating may, in the first instance, be considered as a prolongation of b' till it cuts x, together with the corresponding points for the intersection of b'' with x. Thus the values δ_2' and δ_2'' are here in question and they are

$$\delta_2' = 1.97 \text{ cm.}, \qquad \delta_2' - \delta_1' = .05 \text{ cm.}$$

whence

$$\delta_2'' = 1.65 \text{ cm.}, \qquad \delta_1'' - \delta_2 = .09 \text{ cm.}$$

are the conjugate foci for the extreme rays of the grating, respectively, beyond the conjugate focus of the middle or normal rays b, on x. Hence the mean of the extreme rays lies at .07 cm. beyond (greater θ) the normal ray and the λ found in the first instance is too large as compared with the true value for the normal ray.

The datum .07 cm. may be taken as the excess of $2x$, corresponding to the excess of angle for a grating one half as wide and observed on both sides ($2x$), as was actually the case. Finally, since the whole of the grating is not in focus at once a correction less than .07 cm. for $2x$ must clearly be in question. This is quite below the difference of several millimeters brought out in §§ 4 and 6.

To make this point additionally sure and avoid the assumption of the last paragraph, we will compute the conjugate focus of the central ray (different angles θ) on the b' focal plane parallel to the grating and to x and on the b'' focal plane parallel to x. The computation is simpler if the central ray is thus focused, than if the extreme rays are focused on the x plane. The distance apart will be

$$\delta_3' = g - b' \cos(\theta + \theta')(\tan(\theta + \theta') - \tan\theta),$$
$$\delta_3'' = g - b'' \cos(\theta - \theta'')(\tan\theta - \tan(\theta - \theta'')).$$

Inserting the results for θ, θ_1', θ_1'', b', b'', g,

$$\delta_3' = .06, \quad \delta_3'' = -.04.$$

Both the b foci thus correspond to large angles. Their mean, however, may be considered as vanishing on the intermediate x plane.

Thus it is clear that the effect of focusing is without influence on the diffraction angle and much within the limits of observation. It is therefore probable that the residual discrepancy in the three methods is referable to a lateral motion of the slit itself due to insufficient symmetry of the slides *AA* and *BB* in the above adjustment. This agrees, moreover, with the residual shift observed in the case of parallel rays in § 8.

Brown University,
Providence, R. I.